THE PERFORMANCE OF A WAVE ENERGY CONVERTER IN SHALLOW WATER

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ABSTRACT

The effect of water depth on the power capture of surging point absorber-type wave energy converters is investigated. It is shown that power capture is typically larger in shallow water due to an increase in the surge wave force that more than compensates for the reduction in incident wave power due to seabed friction. It is also shown that this surge wave force is closely related to the increase in horizontal water particle amplitude that occurs when the non-dimensional water depth, *kh*, is less than approximately 1.0.

INTRODUCTION

This paper investigates the power capture of a point absorber-type wave energy converter (WEC) moving in surge/pitch when in deep and shallow water. A point absorber is small relative to the incident wave length, but is capable of capturing wave energy across a frontage larger than its own dimensions (Evans 1980). For this reason the hydrodynamics and performance of point absorbers has attracted significant interest in the development of wave energy converters (Falnes & Budal 1978; French & Bracewell 1985; Neilsen & Plum 2000; McCabe et al. 2003).

Wave energy converters sited in deep water have a number of potential advantages over devices sited closer to the coastline, with the most obvious advantage being a higher average incident wave power compared to a nearshore or shoreline site along the same stretch of coastline. A typical site, which has been extensively monitored and used for wave energy resource calculations, is the South Uist site located on the Western coast of Scotland. At this location, using Waverider buoy data, the average incident power at 100 metres water depth is 65 kW/m, reducing to 48 kW/m at a depth of 45 metres, 34 kW/m at a depth of 25 metres and 15 kW/m at a depth of 15 metres (The Department of Energy 1992). However, a wave refraction analysis implied that the Waverider buoy deployed at a depth of 15 metres was located at a ‘cold spot’ and the analysis suggests that a more representative average incident power at this depth would be 22 kW/m. However, these average incident wave power values include highly energetic sea-states (>2000 kW/m) that will make little contribution to the average power capture because either the device has to shut down during these events, or the maximum plant rating will limit the power capture. These events add significantly more to the average offshore wave energy than they do to the average nearshore wave energy, so that the difference in available incident wave energy between deep and shallow water is less significant than the figures presented above suggest.

However, whilst a nearshore site will have a lower average incident wave power, it does possess a number of attractive characteristics. A nearshore location will reduce both the cost and power losses in the cable bringing power back to shore. A nearshore location may also reduce the costs of installation and maintenance; a shorter distance from the harbour to device reduces travelling time and increases plant availability by utilisation of smaller weather windows for repair and maintenance. With operations and maintenance costs accounting for perhaps 40% of the net present cost of a wave energy converter this may be a significant issue (The Royal Academy of Engineering 2004). Shallow water also filters out the largest waves, potentially reducing the maximum loads required for survival, although placement within the breaking wave zone may negate this advantage and may even cause the maximum loads experienced to increase.

Without detailed engineering studies it is difficult to compare devices in deep and shallow water, indeed device designs are often only suitable for deployment in deep or shallow water making direct comparisons difficult. However, a general perception would be that for similarly sized devices, the power capture for an offshore device would be significantly larger than the power capture for a nearshore device due to the larger incident wave energy. This paper examines this perception with respect to surging point absorbers and concludes that in many cases power capture depends more on the incident wave force than the incident wave power. The effect of this is that power capture may be higher in shallow water even if there is a reduction in incident wave power due to seabed friction and wave breaking.

THE PERFORMANCE OF SURGING POINT ABSORBERS

It has been shown (Evans 1985) that the maximum power capture, P_{max}, for a surging point absorber depends only on the incident wave power, P_i, and wavelength, λ, as given in Equation (1).

$$P_{\max} = \frac{\lambda}{\pi} P_i \tag{1}$$

Using Equation (1) for a North Atlantic site with an average incident wave power of 50 kW/m and an energy period of 10 seconds, which equates to a wavelength of approximately 150 metres, results in a maximum average power capture of 2.5 MW? The projected power capture of a small wave energy converters has never approached this average power capture, which indicates that a more appropriate analysis of performance is required.

To achieve the maximum power capture the wave energy converter must satisfy three conditions. Firstly, the amplitude of motion of the wave energy converter must be such that the net power is maximised. The maximum amplitude of motion will be constrained by moorings, rotation limits or the dynamics of the system. If the maximum amplitude of motion is smaller than the optimum amplitude of motion for maximum power capture then the power capture will be reduced. Secondly, the velocity of the wave energy converter must be in-phase with the wave force to ensure that the waves are always doing work on the device. Without perfect complex conjugate control there will be a reduction in the power capture. Thirdly, there must be zero parasitic losses. Parasitic losses are typically due to viscous effects, such as the generation and shedding of vortices.

The effect on power capture when the maximum amplitude of motion is less than the optimum amplitude of motion has been previously investigated (Evans 1985). Using the ratio of amplitude of motion to optimum amplitude of motion, r, then the power capture, P, is given by Equation (2)

$$P = P_{\max}(2r - r^2) \tag{2}$$

However, the maximum power capture can also be given as $P_{\max} = F^2/_{8B}$, where F is the wave force and B is the radiation damping coefficient. Also noting that the optimum amplitude of motion, x_0, can be given by, $x_0 = F/_{2\omega B}$, then Equation (2) can be re-written as

$$P = \tfrac{1}{2} F \cdot \omega \cdot x(1 - \tfrac{1}{2} r) \tag{3}$$

Equation (3) shows that in cases where the motion is highly constrained, i.e. $r \rightarrow 0$, then the power capture is proportional to the wave force. This is an unsurprising result and effectively implies that for small bodies the radiated power is negligible. An equivalent result to Equation (3) has also been used by Budal and Falnes to argue that the highest power capture per unit volume of a heaving body will be achieved when the body is much smaller than the wavelength (Budal & Falnes 1980); the buoy diameter proposed by Budal and Falnes was 6 metres. However the key significance of Equation (3) in this analysis is that it demonstrates that for highly constrained small bodies the power capture depends primarily on the incident wave force, not the incident wave power.

THE SURGE WAVE FORCE ON SMALL BODIES

To understand the magnitude of the surge wave force on small bodies it is useful to consider the long-wave approximation(Simon & Hulme 1985). A long-wave first order approximation shows that the surge wave force depends on the horizontal water particle acceleration, $\omega^2\eta$, the body displaced mass, M, and the added mass, M_a, as shown in Equation (4)

$$F = (M + M_a)\omega^2\eta \tag{4}$$

Assuming that the added mass is unaffected by the water depth, h, then for the same wave in deep and shallow water, the ratio of surge wave forces is simply equal to the ratio of horizontal water particle displacement, R. Figure 1 shows the ratio of the horizontal water particle displacement at the surface derived from linear wave theory as the wave moves from deep to shallow water. The increase in horizontal displacement is due to the combination of the change in wave height due to shoaling, together with the enhanced elliptical particle motion that occurs in shallow water.

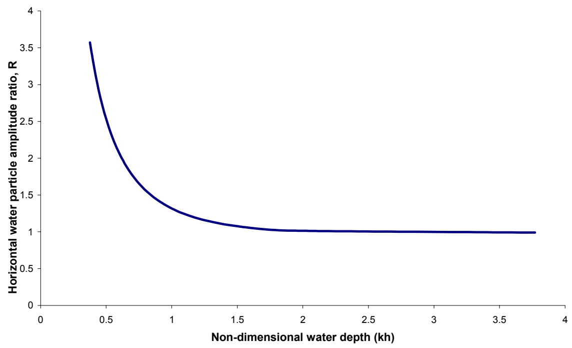

Figure 1 Effect of water depth on horizontal water particle motion

The accuracy of this approximation can be tested using linear hydrodynamic numerical models, such as WAMIT®. Figure 2 compares the surge wave force ratio calculated for hemispheres with different relative radii, *R/h*, with the approximation based on the horizontal water particle motion. This figure shows that the approximation is very good except for *R/h* = 1, where the hemisphere radius is equal to the water depth. In this case the added mass of the hemisphere is increased by the the presence of the seabed, increasing the surge wave force ratio. WAMIT models of truncated cylinders and flaps have shown a similarly high degree of correlation between the calculated surge force ratio and the increase in horizontal water particle motion.

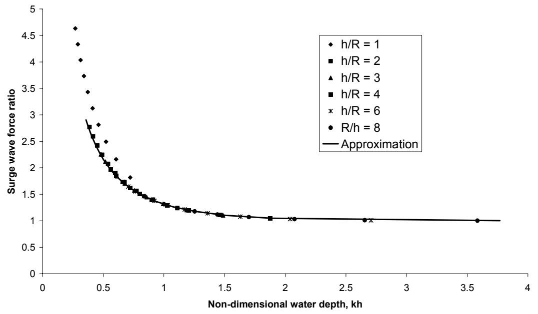

Figure 2: Comparison of approximation with surge wave force ratio for a hemisphere

For example, a 10 second wave in a water depth of 12 metres equates to a non-dimensional water depth, *kh*, of approximately 0.75, resulting in a horizontal water particle amplitude ratio of 1.50 (produced by a wave shoaling coefficient of 0.96 and a horizontal water particle motion equal to 1.56 times the wave height). For this wave a 12 metre diameter floating hemisphere has a surge wave force ratio of 1.50; that is, if no energy is lost as the water depth decreases, the surge wave force experienced in a water depth of 12 metres is 1.50 larger than the force the floating hemisphere would experience in deep water.

The above analyses have shown that the wave force increases with a reduction in water depth and that this increase in wave force is primarily associated with the increased horizontal water particle motion. It has also been shown that the increase in surge wave forces occurs in a water depth that can be readily exploited (12 metres) by wave energy converters experiencing typical North Atlantic waves (10 second wave period).

THE EFFECT OF WATER DEPTH ON INCIDENT WAVE POWER

The incident wave power decreases in shallow water because of energy losses due to seabed friction and wave breaking (Department of Energy 1992). The amount of energy lost depends on both the bathymetry and the wave characteristics. A gently shelving seabed will increase the amount of energy lost because of seabed friction due to the increased length over which there is significant water particle motion at the seabed creating shear. A rough and/or highly vegetated seabed will similarly increase the amount of energy lost. Energy losses due to seabed friction will also increase with large and long period waves because of increased water particle motion at the seabed. Signficant energy loss due to wave breaking occurs when the wave heights are greater than approximately 0.5 of the water depth and thus is uncommon except in very high-power seas or in very shallow water.

The complexity of energy loss, as waves progress from deep to shallow water, makes it difficult to generalise about the effect of water depth on incident wave power. However, an indication of the effect of water depth on incident wave power can be obtained by analysing resource studies. The UK shoreline wave energy resource study (Department of Energy 1992) provides data for the incident wave power in deep water, on the 40 metre contour, the 20 metre contour and the 10 metre contour for approximately 60 distinct sea-states different locations around the UK, which makes it particularly useful.

Figure 3 shows how the reduction in incident wave power from the 40 metre contour to the 10 metre contour is influenced by the incident wave power for the Islay data set (the sea-states have been sorted into 10 kW/m bins to reduce the scatter shown). Although there are a number of anomalies, this clearly shows that the percentage reduction in incident power is larger when there is more incident power and that for incident powers of less than 100 kW/m the reduction is typically less than 10%; this represents the vast majority of sea-states likely to occur at this site. A similar analysis has been performed for data from the South-west Approaches with similar characteristics.

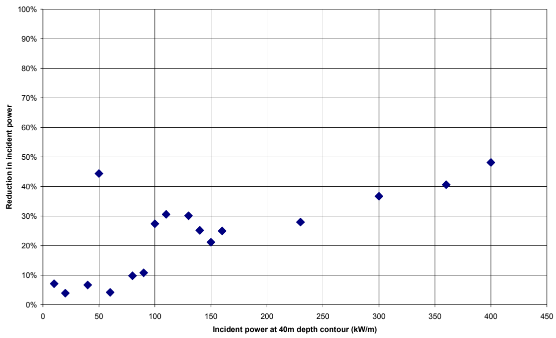

Figure 3: Reduction in incident wave power from the 40 metre to 10 metre contour at Islay

This short analysis has shown that for the majority of incident wave power (< 100 kW/m) there is only a modest power loss (~10%) due to seabed friction and wave breaking, which is significantly less than the reduction in average annual incident power.

THE RELATIVE POWER CAPTURE OF SURGING WEC'S

The combined effect of an increase in the wave force and a reduction in incident wave power can be investigated by comparing the predicted power capture of a nominal surface-piercing flap-type WEC when deployed in deep (h = 50 metres) and shallow (h = 12 metres) water. The nominal flap analysed is 12.0 metres wide, extends 10.5 metres below the water surface and is 1.0 metre thick. The natural period of the flap is 12.0 seconds and power is extracted using a linear damper. Phase control is not used to maximise the power capture, however the linear damping coefficient is modified to maximise the power capture at each incident wave frequency. The loss of power due to bottom friction from the 50 metre to 12 metre contour is modelled as being independent of wave frequency and a conservative value of 20% has been used.

The power capture of the WEC is estimated using a linearised model of its dynamics, using hydrodynamic coefficients produced by WAMIT. Losses due to vortex shedding are modelled using a Lorentz linearisation (Terra et al. 2005), which replaces the non-linear, quadratic, drag coefficient, C_D, with a linear coefficient, B_V, so that the same energy dissipation occurs in each representation.

$$B_V = {}^{8}\!/_{3\pi} C_D \cdot \omega |X| \qquad (5)$$

Forced oscillation of a model flap in a wave-tank demonstrated a quadratic drag force, F_D, given by Equation (6)

$$F_D = C_D \cdot \omega^2 |X| X \qquad (6)$$

where X is the amplitude of motion and C_D is the drag coefficient (Folley 2004). At full scale the drag coefficient is 388 kNs^2/m^2.

Thus, the frequency domain equation of motion is given by

$$F = \lfloor (M_0 + M_a)(\omega_N^2 - \omega^2) + j\omega(B + B_V + \lambda) \rfloor X \qquad (7)$$

where M_0 is the flap's mass, ω_N is the flap's natural frequency, B is the hydrodynamic damping coefficient and λ is the applied damping coefficient.

The power capture, P, is calculated using the well-known equation (Evans 1985)

$$P = \tfrac{1}{2} \lambda \omega^2 |X|^2 \qquad (8)$$

For each wave frequency the power capture is maximised by optimisation of the applied damping, whilst ensuring that the drag losses are correctly modelled using equations (5) and (7).

The power capture of the flap in water depths of 50 metres and 12 metres for a deep-water incident wave amplitude of 1.0 metres is shown in Figure 4.

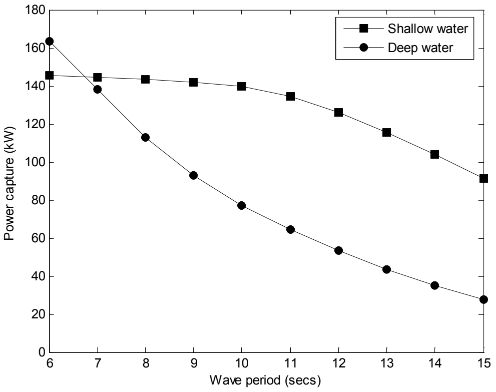

Figure 4: Relative performance of a flap-type WEC in deep and shallow water

Figure 4 shows that, except for the shortest wave periods, this WEC has a significantly higher power capture in the shallower water, even though the incident wave energy has been reduced by 20%. In the shortest waves the increase in horizontal water particle motion does not compensate for the loss of power resulting in the reduction in power capture seen. However, it is likely that power loss due to bottom friction would be smaller for short wave periods because the amplitude of particle motion at the seabed is smaller. Thus the power capture may be higher for all wave periods when the WEC is located in shallow water.

Perhaps surprisingly, the power capture is more than doubled for wave periods above 11 seconds, which is greater than the increase in wave force. However, unlike in Equation (3), where the ampltiude of motion is rigidly and severely constrained and the power capture is proportional to the wave force, the optimum amplitude of body motion for this WEC also increases with wave force. This results in an increase in power capture greater than the increase in wave force. The increased optimum amplitude of motion and applied damping coefficient are shown in Figure 5 and Figure 6 respectively.

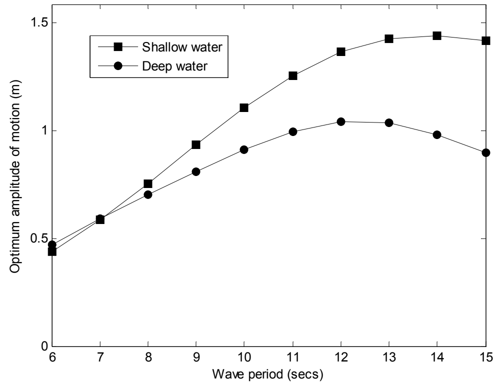

Figure 5: Effect of water depth on optimum amplitude of motion

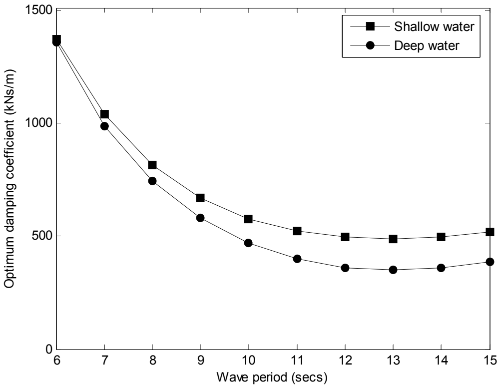

Figure 6: Effect of water depth on optimum damping coefficient

Figure 4 also shows that the power capture of the WEC in shallow water varies much less with wave period than it does in deep water. This broader bandwidth is a potentially useful characteristic of a WEC because it should result in a more consistent power output and thus a better load factor (Whittaker & Folley 2005). Because the surge wave force is approximately proportional to the horizontal water particle acceleration there is a dramatic reduction in wave force at low frequencies (acceleration is proportional to frequency squared). However, this is partially offset in shallow water due to the increased horizontal water particle amplitude, which is larger at low frequencies, resulting in less variation of the surge wave force.

Although the flap has a natural period of 12.0 seconds, Figure 4 does not show an increase in power capture at this period, which would be expected. This is because the resonant peak has been suppressed by the drag losses. Figure 7 shows the power capture with the drag coefficient reduced to 100 kNs^2/m^2, which clearly shows the increase in performance close to the flap's resonant period.

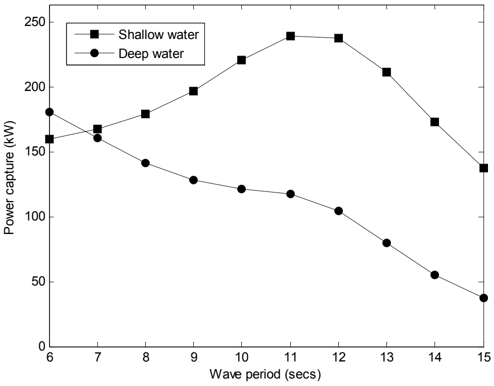

Figure 7: Power capture of a flap-type WEC with a reduced drag coefficient

Figure 8 shows how the power capture is affected if the simple linear damper control is replaced with complex-conjugate control. As would be expected the power capture is increased (compared to Figure 4), but the general relationship between the performance in deep and shallow water remains similar.

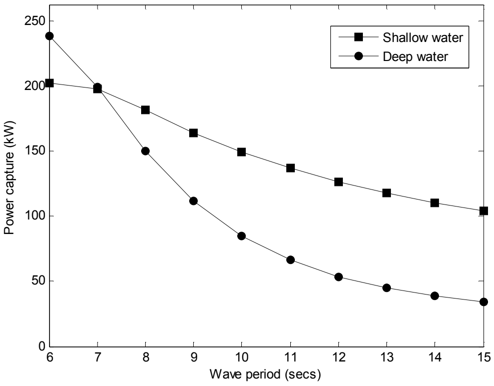

Figure 8: Power capture of a flap-type WEC with complex conjugate control

In addition to the modifications of the model described above, changes to the incident wave amplitude has been found to not affect the general relationship between the WEC power capture in deep and shallow water. Increasing the incident wave power loss as it progresses from deep to shallow water clearly influences the relationship between the two power captures, however the power loss needs to increase to 50% before the average power captures are approximately equal as shown in Figure 9.

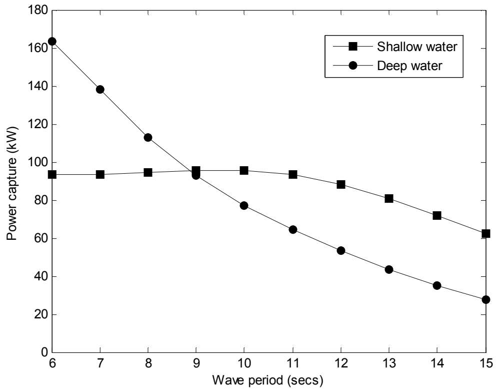

Figure 9: Relative performance of a flap-type WEC with a 50% wave power loss between deep and shallow water

CONCLUSIONS AND FURTHER WORK

The analysis above has shown that the power capture of small surging WEC's can be increased by locating them in shallow water, rather than deep water, and that this more than compensates for typical amounts of incident wave power reduction due to bottom friction. It has also been shown that this increased power capture is due to an increase in the surge wave force associated with the larger horizontal water particle motions that occur in shallow water.

These conclusions are specific to small surging WEC's and should not be used to infer that shallow water would be a beneficial location for other devices. Moreover, engineering aspects such as mode of operation, configuration, mooring arrangement, survival strategies, etc., all tend to define the optimum water depth for a WEC's deployment; that is, the design of the WEC defines the water depth. However, the above analysis does show that WEC's deployed in the nearshore region will not necessarily have a lower power capture than those deployed in deeper water and therefore should not be summarily dismissed.

This paper reports on an ongoing research study, and further work is required in two main areas. Firstly, further work is required in analysing the reduction in wave power as it approaches the coast. A large amount of scatter is evident in Figure 3 indicating that the processes are complex. A better understanding of how the reduction in wave power is influenced by the bathymetry and incident wave characteristics will enable the conclusions to be provided with a greater level of confidence and validity. A greater level of insight will also help to identify whether particular coastlines are especially suited to nearshore WEC's. Indeed, by breaking the rigid link between annual average incident wave power and power capture other sites may be more suitable for small surging WEC's. In particular, the relationship between surge wave force and horizontal water particle acceleration implies that sites that experience shorter, steeper waves may be more suitable.

Secondly, the comparative performances of the WEC in deep and shallow water have been estimated using a quasi-linear analysis. Further confidence in the results will be obtained by modelling of the WEC's in the wave-tank, with the models being subjected to realistic wave-trains to ensure that any conclusions remain valid for typical sea-states.

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